

Optimising Stop Naturalness

Chris Wymant

*Institute for Particle Physics Phenomenology
Department of Physics, Durham University
Durham DH1 3LE, United Kingdom*

September 7, 2012

Abstract

In supersymmetric models the average stop mass M_S is well known to contribute both to the lightest Higgs boson mass m_h and to the level of unnaturalness in radiative electroweak-symmetry breaking (REWSB). The same is true of the stop trilinear mixing term A_t , though its role in naturalness receives rather less attention. We address this issue in this note by using Lagrange constrained maximisation to optimise radiative corrections to m_h for a constant REWSB term $\delta m_{H_u}^2$ in the MSSM. The ratio A_t^2/M_S^2 is robustly predicted to be close to maximal mixing: $\sim 5 - 6$. Experimental uncertainties alone (on the top and Higgs masses) may prevent any inference of the M_S associated with a given Higgs mass, mixing parameter and $\tan\beta$. We also emphasise that a fair comparison of how *easily* different models can reach a particular m_h should attempt to maximise m_h with respect to some upper-bounded measure of naturalness, and not simply compare stop masses. For example with respect to the leading-log $\delta m_{H_u}^2$, maximal-mixing scenarios are as natural as no-mixing scenarios with stops twice as heavy.

1 Introduction

Radiative electroweak-symmetry breaking (REWSB) in the Minimal Supersymmetric Standard Model (MSSM) occurs when the mass matrix for the CP-even Higgs scalars develops a negative eigenvalue. For moderate-to-large $\tan\beta$ this is due to the soft SUSY-breaking mass-squared for the H_u superfield, $m_{H_u}^2$, driven by radiative corrections to be more negative than the supersymmetric Higgs mass-squared $|\mu|^2$. A non-zero Z mass results, according to

$$-\frac{1}{2}M_Z^2 = m_{H_u}^2 + |\mu|^2 + \mathcal{O}((\tan\beta)^{-2}). \quad (1)$$

For Eq. (1) to be free from significant cancellation, and REWSB to be ‘natural’, the radiative corrections to $m_{H_u}^2$ must not be dramatically larger than the Z mass squared¹.

The stop mass (together with the stop mixing) features prominently in the radiative corrections to $m_{H_u}^2$ and thus we are led to consider light stops for naturalness. The impressive constraints on the squarks of the first two generations now with $\sim 5\text{fb}^{-1}$ of data (see e.g. [4, 5])

¹ Note that the cancellation between $m_{H_u}^2$ and $|\mu|^2$ can still be small with a large radiative correction $\delta m_{H_u}^2$, if large cancellation occurs instead between $\delta m_{H_u}^2$ and the high-scale value of $m_{H_u}^2$; *a priori* this is theoretically unjustifiable. Such situations may be natural in the spirit of Barbieri-Giudice [1], as M_Z can be somewhat insensitive to the UV parameters, but will have high fine-tuning as measured à la Kitano-Nomura [2] or Baer *et al.* [3], as the required Z mass still results from large terms cancelling.

are known to be considerably relaxed for squarks of the third generation, due to direct production cross-sections suppressed by PDFs and the less distinctive final states that may result (e.g. being too similar to Standard Model top backgrounds [6]). This weakening of the bounds for stops and sbottoms is of course only useful for naturalness in the context of models where the third generation squarks are lighter than those of the first two. [7] pointed out that this effect ought be a feature of the mediation of SUSY breaking rather than an RG effect, since RG-induced splitting of the squark masses also drives the running $m_{H_u}^2$ – precisely what we are trying to avoid. (An exception to this would be a heavy Right-Handed sbottom at large $\tan\beta$, allowing the Left-Handed stop to run lighter than the LH sup and scharm without affecting $m_{H_u}^2$ or the Z mass condition (1).) See [7, 8] and references therein for discussions of light third-generation squarks. Recent discussions of stop limits and discovery potential can be found in [9–20].

The stop mass and stop mixing also contribute radiatively to the physical mass of the lightest CP-even Higgs boson h . At tree level m_h is bounded from above by $M_Z \cos 2\beta$, and saturates this bound in the ‘decoupling limit’ where the pseudoscalar A is appreciably heavier than M_Z . In this limit h also has Standard Model-like couplings, which are favoured by the roughly Standard Model-like strengths seen in different channels² for the 126 GeV resonance [23, 24] assumed to be a Higgs boson. It has long been known that, in the MSSM at least, some substantial combination of stop mass- and stop mixing-induced corrections to m_h is needed to lift it above the LEP lower bound of 114.4 GeV. A ~ 126 GeV Higgs requires these corrections to be even more substantial, with correspondingly worse implications for naturalness. The interplay of parameters for such a Higgs mass when looking agnostically at the MSSM has been investigated in [3, 6, 25–40]. Even remaining in the field of supersymmetry many more works have shown the implications of such a Higgs mass in particular models of SUSY breaking, in extensions of the MSSM, or else have focused predominantly on issues relating to the decays of the Higgs into different final states.

In this work we focus on the effects of the stop sector on the physical Higgs mass m_h and on the REWSB term $\delta m_{H_u}^2$; specifically on the relationship between them. We start by recognising that in the literature there is generally an asymmetry in the treatment of the average stop mass M_S and the stop mixing parameter $x \equiv A_t^2/M_S^2$. The former is raised to sufficiently large values to help with m_h , tempered by a desire not to make it too large for naturalness’ sake. The latter, where possible (i.e. outside of $A_t \approx 0$ models), is then freely chosen to boost m_h . However this too contributes to unnaturalness. Perhaps a smaller mixing parameter should have been chosen, with heavier stops instead. Would this have been as good? We approach this problem analytically, neglecting effects from outside of the stop sector, using Lagrange constrained minimisation to reveal the optimum balance between M_S and x . A more approximate method (and transparent result) is laid out in section 2. Higher-order complications are considered in section 3. Section 4 contains a brief discussion and conclusions.

2 Leading-Order Analysis

The one-loop beta function of $m_{H_u}^2$ [41] is:

$$16\pi^2 \frac{d}{dt} m_{H_u}^2 = 6y_t^2(m_{\tilde{Q}_3}^2 + m_{\tilde{u}_3}^2 + A_t^2) + 6y_t^2 m_{H_u}^2 - 6g_2^2 M_2^2 - \frac{6}{5}g_1^2 M_1^2 + \frac{3}{5}g_1^2 \text{Tr}[Y_{\tilde{f}} m_{\tilde{f}}^2], \quad (2)$$

² The diphoton ‘excess’ is downplayed in [21], where QCD uncertainties in gluon fusion show the discrepancy is only 1σ , and also in [22], based simply on an alternative analysis of the coupling data.

where $t = \log(Q/\Lambda)$, with Λ the high/mediation scale at which the soft SUSY-breaking mass terms are generated. One can roughly neglect the terms of the second line³; keeping only the large stop-sector terms, taking these to be constant and integrating gives the oft-quoted Leading-Log (LL) expression

$$\delta m_{H_u}^2 \approx -\frac{3}{8\pi^2} y_t^2 (m_{\tilde{Q}_3}^2 + m_{\tilde{u}_3}^2 + A_t^2) \log\left(\frac{\Lambda}{M_S}\right), \quad (3)$$

at a scale M_S – the scale at which Eq. (1) holds most accurately, [43–45].

The dominant radiative correction δm_h^2 to the tree-level Higgs mass-squared $m_{h,\text{tree}}^2 = M_Z^2 \cos^2 2\beta$ (taking the decoupling limit) is

$$\delta m_h^2 \approx \frac{3}{4\pi^2} \frac{m_t^4}{v^2} \left(\log\left(\frac{M_S^2}{m_t^2}\right) + \frac{X_t^2}{M_S^2} \left(1 - \frac{X_t^2}{12M_S^2}\right) \right) \quad (4)$$

with $v = 174$ GeV and $X_t = A_t - \mu \cot \beta$.

Firstly we note that through Eq. (1), the required $|\mu|$ depends on the unknown high-scale value of $m_{H_u}^2$ as well as its radiative corrections. However, a) the aim for natural SUSY is $|\mu|/(100 \text{ GeV}) \lesssim$ a few, b) a large Higgs mass ~ 126 GeV needs⁴ $\tan \beta \gtrsim \mathcal{O}(5)$, and c) later we will arrive at $A_t \gtrsim \mathcal{O}(1 \text{ TeV})$. Thus we motivate $X_t \approx A_t$ without knowing the value of μ .

Secondly, we see that while the Higgs mass depends only on an average stop mass M_S , $\delta m_{H_u}^2$ depends on both M_S and the precise linear combination $m_{\tilde{Q}_3}^2 + m_{\tilde{u}_3}^2$. We then need to choose exactly how we define this average mass M_S . Often this is taken to be a geometric mean. The minimum $(m_{\tilde{Q}_3}^2 + m_{\tilde{u}_3}^2)$ for constant $(m_{\tilde{Q}_3}^2 \times m_{\tilde{u}_3}^2)^{1/2}$ then provides weak motivation for $m_{\tilde{Q}_3}^2 = m_{\tilde{u}_3}^2$. If instead the linear average $M_S^2 \stackrel{?}{=} \frac{1}{2}(m_{\tilde{Q}_3}^2 + m_{\tilde{u}_3}^2)$ is chosen, then only this combination appears in both $\delta m_{H_u}^2$ and the Higgs mass m_h ; the orthogonal combination is entirely free. A further alternative would be to take an average of the mass eigenvalues $m_{\tilde{t}_{1,2}}$: the dependence of $\delta m_{H_u}^2$ on the underlying parameters $m_{\tilde{Q}_3}^2, m_{\tilde{u}_3}^2, A_t$ then shifts very slightly but becomes a complicated function. We can appeal to the limit $\log(\Lambda/M_S) \gg \log(m_{\tilde{t}_2}/m_{\tilde{t}_1})$, in which the former log and thus $\delta m_{H_u}^2$ no longer depends on how we define the average stop mass M_S . All considered, it seems a safe conclusion that we can make deductions about M_S from naturalness and the Higgs mass, but the splitting between the stops is unknown and free⁵. Henceforth it is to be understood that the aforementioned linear average is implied, so that equations (3) and (4) depend on only two soft mass scales: M_S and A_t .

We are now in a position to use Lagrange constrained minimisation: the solution of

$$\frac{\partial}{\partial(M_S^2)} (\delta m_h^2 - \lambda \delta m_{H_u}^2) = \frac{\partial}{\partial(A_t^2)} (\delta m_h^2 - \lambda \delta m_{H_u}^2) = 0, \quad (5)$$

³ The effect of $m_{H_u}^2$ on its own running is small if the Leading-Log approximation is valid (i.e. (one-loop factor) $\times \log(\Lambda/M_S) \lesssim 1$). Then, since the overall radiative correction must be substantial enough to turn $m_{H_u}^2$ negative, the $m_{H_u}^2$ term in the beta function must be appreciably smaller than the other terms. The electroweak couplings we neglect, dominated as they are by y_t^2 . The wino term can be important [38], but here we will be differentiating with respect to stop-sector terms, so this effect drops out. While the trace term is a sum over all scalars, it couples only through g_1 and is ‘relatively small in most known realistic models’ [41]. For example it vanishes at the high scale in all models of General Gauge Mediation [42], and all models with universal scalar masses (such as minimal SUGRA) since $\text{Tr}[Y] = 0$. Furthermore the running of the trace is proportional to the trace itself.

⁴ Unless one enters the realm of split SUSY $M_S \gtrsim \mathcal{O}(10^{4.5} \text{ GeV})$, [46].

⁵ Eq. (3) as written makes clear the insensitivity of $\delta m_{H_u}^2$ to the splitting $|m_{\tilde{Q}_3}^2 - m_{\tilde{u}_3}^2|$ and thus to the splitting of the mass eigenvalues. When (3) is re-written in terms of the physical masses, there is a term proportional to $(m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2)^2 \sin^2 \theta_{\tilde{t}}/v^2$ which appears to favour minimal splitting; this inconsistency is resolved by noting that as $|m_{\tilde{Q}_3}^2 - m_{\tilde{u}_3}^2|$ grows, the stop mixing angle $\theta_{\tilde{t}}$ decreases. It is interesting that from the point of view of unconstrained splitting, null searches for sbottoms do not translate into bounds on the lightest stop, c.f. [6].

where λ is the unspecified Lagrange multiplier, is the most natural ratio of A_t^2 to M_S^2 , with the overall scale of one of these two parameters freely chosen thereafter. Explicitly, with the one-loop δm_h (4) and the Leading-Log δm_h^2 (3):

$$x_{\text{natural}} \equiv \left(\frac{A_t^2}{M_S^2} \right)_{\text{natural}} = 2 + \sqrt{4 + \frac{6(L-2)}{L-1}}, \quad (6)$$

with $L = \log(\Lambda^2/M_S^2)$. The solution is real for $L > \frac{8}{5}$, asymptotes to $2 + \sqrt{10} \approx 5.16$ as $L \rightarrow \infty$, and is already 5 for $L = 7$ (i.e. $\Lambda/M_S = 33$) – thus it is essentially constant over phenomenologically interesting mediation scales. That the optimal x should be *less* than the ‘maximal mixing’ value $x = 6$ is obvious: decreasing it from 6 to $6 - \delta$ reduces the physical Higgs mass by $\mathcal{O}(\delta^2)$ but increases naturalness by $\mathcal{O}(\delta)$. That it should be *close* to six is easily understood: using the (logarithmic) stop mass term to boost the Higgs mass requires exponentially heavy stops and thus exponentially bad fine-tuning; whereas the stop mixing term contribution to m_h can be large even for very small A_t^2 and M_S^2 , provided their ratio is favourable.

3 Higher-order Effects

3.1 Finding x_{natural}

Higher order effects of the stop on the physical Higgs mass can be taken into account with the two-loop expression of [47]:

$$\begin{aligned} \delta m_h^2 &= \frac{3}{4\pi^2} \frac{m_t^4}{v^2} \left[\frac{1}{2} \tilde{X}_t + (1+D)T + \epsilon \left(\tilde{X}_t T + T^2 \right) \right], \quad (7) \\ \text{with } m_t &= \frac{M_t}{1 + \frac{4}{3\pi} \alpha_3(M_t)}, \\ \alpha_3(M_t) &= \frac{\alpha_3(M_Z)}{1 + \frac{23}{12\pi} \alpha_3(M_Z)}, \\ T &= \log \frac{M_S^2}{M_t^2}, \\ D &= -\frac{M_Z^2}{2m_t^2} \cos^2 2\beta, \\ \tilde{X}_t &= \frac{2A_t^2}{M_S^2} \left(1 - \frac{A_t^2}{12M_S^2} \right), \\ \text{and } \epsilon &= \frac{1}{16\pi^2} \left(\frac{3}{2} \frac{m_t^2}{v^2} - 32\pi \alpha_3(M_t) \right) \end{aligned}$$

(which also includes the smaller, soft-mass independent, one-loop D -term $\mathcal{O}(M_Z^2 m_t^2)$ of [48]). The maximisation, Eq. (5), goes through exactly as before. The solution is the positive root of the following equation (which recovers Eq. (6) as $D, \epsilon \rightarrow 0$)

$$\begin{aligned} [1 + 2\epsilon T + L(-1 + \epsilon - 2\epsilon T)] x_{\text{natural}}^2 \\ + 4[-1 - 2\epsilon T + L(1 - 3\epsilon + 2\epsilon T)] x_{\text{natural}} \\ - 6[2 + 4\epsilon T + L(-1 + D - 2\epsilon T)] = 0 \quad (8) \end{aligned}$$

We show the variation of this solution with M_S in Fig. 1; dependence on $\tan \beta \in [5, 45]$ and the top mass uncertainty is negligible.

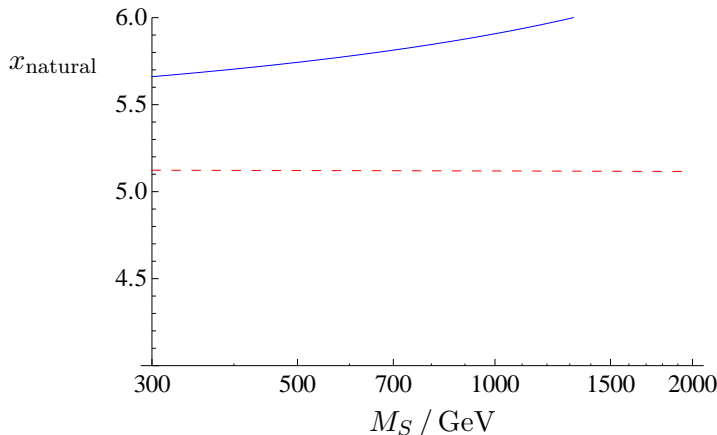


Figure 1: The most natural ratio $x \equiv A_t^2/M_S^2$ obtained from maximising the Higgs mass at one-loop (red, dashed) and two-loop (blue, solid) for constant electroweak symmetry breaking term $\delta m_{H_u}^2$, as a function of the average stop mass M_S .

Two other approaches are trivially equivalent to using Eq. (5) to find x_{natural} . Firstly, one could invert the $\delta m_{H_u}^2$ expression to find the function $M_S(x)|_{\delta m_{H_u}^2}$ for how the stop mass must vary as a function of x in order to keep $\delta m_{H_u}^2$ constant: from Eq. (3) this monotonically decreasing function is

$$M_S(x)|_{\delta m_{H_u}^2} = \Lambda \exp\left(\frac{1}{2} W_{-1}\left(\frac{-16\pi^2 \delta m_{H_u}^2}{(2+x)\Lambda^2}\right)\right) \quad (9)$$

where $W_{-1}(\dots)$ is the lower branch of the Lambert W function. The one-parameter function $\delta m_h^2(x, M_S(x)|_{\delta m_{H_u}^2})$ then gives the range of Higgs masses possible for a given $\delta m_{H_u}^2$; the *maximum* occurs at x_{natural} .

Secondly, one could invert the δm_h^2 expression to find the function $M_S(x)|_{\delta m_h^2}$ for how the stop mass varies as a function of x for a constant Higgs mass. This function is easily obtained from Eq. (7) which is a quadratic equation in $\log(M_S^2/M_t^2)$; we plot it in the left panel of Fig. 2. The one-parameter function $\delta m_{H_u}^2(x, M_S(x)|_{\delta m_h^2})$ then gives the range of $\delta m_{H_u}^2$ possible for a given Higgs mass, depending on the amount of stop mixing (x) one uses to achieve that Higgs mass. The *minimum* occurs at x_{natural} . We plot this in the right panel of Fig. 2, normalised to $\frac{1}{2}M_Z^2$ for a transparent indication of fine-tuning.

The different colours (line styles) in Fig. 2 correspond to different m_h ($\tan\beta$), see the caption. We see that the greater the m_h we require (and the lower $\tan\beta$ is), the larger x must be to even find a solution: no-mixing scenarios are more limited in the Higgs mass they can reach before the m_h expression (7) breaks down. Indeed even using the program FeynHiggs [49] for a higher order calculation, in the no-mixing $x = 0$ scenario breakdown occurs before one can reach $m_h \sim 126$ GeV and one must resort to a matching of the MSSM on the SM, as noted in [29].

The left panel of Fig. 2 illustrates the obvious fact that the smallest stop mass for a given Higgs mass occurs at exactly maximal mixing $x = 6$. Close inspection of the right panel shows the more subtle point that the lowest fine-tuning occurs at *almost* maximal mixing. We see from the flatness of the curve for $x \in [5, 6]$, however, that the difference between the two is essentially nil.

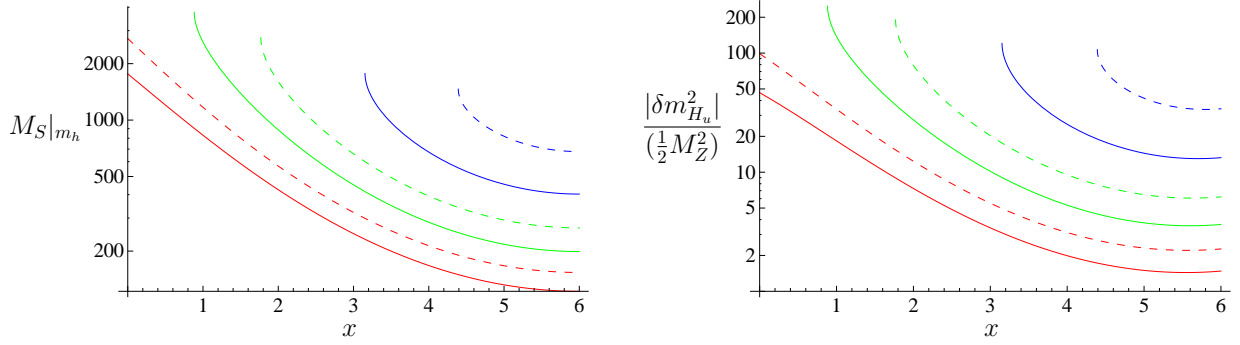


Figure 2: x -axis: $x \equiv (A_t^2/M_S^2)$. The left panel shows the average stop mass M_S required for constant Higgs mass m_h ; the right panel shows the fine-tuning $|\delta m_{H_u}^2|/(\frac{1}{2}M_Z^2)$ that results. Red curves (the lowest two) have $m_h = 115$ GeV, green curves (the middle two) $m_h = 119$ GeV, and blue curves (highest) $m_h = 123$ GeV. Dashed (solid) lines have $\tan \beta = 8$ (30).

3.2 Finding the stop mass?

Varying M_S while keeping $x = x_{\text{natural}}$ fixed traces out the Higgs mass that results in this most natural setting. Of course to go to the full Higgs mass from only the stop radiative corrections one must either neglect the corrections from other sparticles (and so certainly steer clear of the large bottom-Yukawa regime at $\tan \beta \gtrsim \frac{m_t}{m_b}$), or else choose some representative value for all of their masses and calculate their fixed contribution. We do the former in Fig. 3. We will first explain the range of validity of Fig. 3 before discussing the uncertainty arising from the top mass, shown with grey bands.

The Higgs mass expression (7) assumes a single-step decoupling of squarks, thus requiring $m_{\tilde{t}_1} \gtrsim \frac{3}{5} m_{\tilde{t}_2}$ [47] which amounts to a lower bound on M_S for validity of the expression. The lower bound is minimal when $m_{\tilde{Q}_3}^2 - m_{\tilde{u}_3}^2$ (which we have argued can be freely chosen) vanishes; we plot the resulting stop mass eigenvalues also in Fig. 3. $m_{\tilde{t}_1} \gtrsim \frac{3}{5} m_{\tilde{t}_2}$ can be seen to imply $M_S \gtrsim 850$ GeV (this large value being due to the large mixing). (7) also does not contain higher order terms $\mathcal{O}(\log^3(M_S^2/M_t^2))$, giving a corresponding upper bound for its validity. Its accuracy is ~ 2 GeV for $M_S \lesssim 1.5$ TeV [47].

Notice in Fig. 1 that at $M_S \sim 1.3$ TeV, x_{natural} becomes as high as 6 (and takes higher values still for $M_S \gtrsim 1.3$ TeV). This signals a breakdown in our procedure, since the Higgs mass expression is a symmetric function of x about the value 6, but naturalness always favours lower values for given M_S . From Fig. 3, we see that at $M_S \sim 1.3$ TeV the derivative of the Higgs mass with respect to M_S vanishes⁶, which is purely an artefact of the truncated expression. The Lagrange constrained maximisation, (5), is then solved by the Higgs mass *alone* maximised with respect to both of its arguments, with the Lagrange multiplier λ vanishing i.e. the naturalness consideration decouples. Hence the solution is pushed onto exactly maximal mixing. Even higher terms in the Higgs mass expression would be needed to push this breakdown point out to higher stop masses.

The authors of [50], following a similar analysis to [52], take the top mass measurement relevant for calculation of the (Standard Model) Higgs mass to be a combined measurement of the pole mass from the Tevatron, ATLAS and CMS: $M_t = (173.1 \pm 0.7)$ GeV. In [51] it was argued that a more theoretically rigorous approach is to use the measurement of the top pair production cross section, and thence extract $M_t = (173.3 \pm 2.8)$ GeV. We show both cases in

⁶ Note that the derivative of interest is m_h with respect to M_S , with x held constant; in Fig. 3 the latter is *not* constant. However it is varying sufficiently slowly that when we instead hold it exactly constant, the relevant derivative still vanishes at the same point $M_S \sim 1.3$ TeV

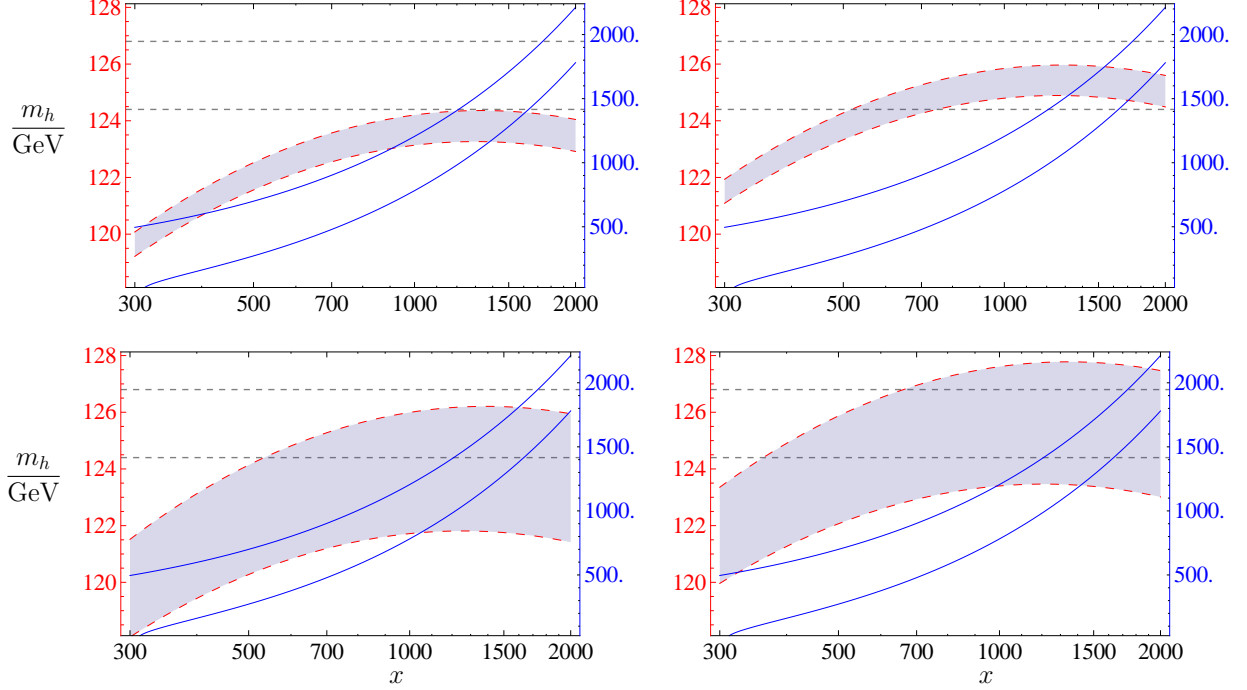


Figure 3: Blue solid lines, right-hand y -axis: the tree-level stop mass eigenvalues $m_{\tilde{t}_{1,2}}$, assuming $m_{\tilde{Q}_3}^2 = m_{\tilde{u}_3}^2$. Red dashed lines, left-hand y -axis: the two-loop expression of [47] for the mass of the lightest CP-even Higgs boson m_h , valid for $850 \text{ GeV} \lesssim M_S \lesssim 1500 \text{ GeV}$. Further dashed lines indicate the lowest m_h compatible with CMS's $m_h = (125.3 \pm 0.4_{\text{stat}} \pm 0.5_{\text{syst}}) \text{ GeV}$ and the highest m_h compatible with ATLAS's $m_h = (126.0 \pm 0.4_{\text{stat}} \pm 0.4_{\text{syst}}) \text{ GeV}$ [23, 24]. Curves are plotted as a function of the average stop mass M_S , with the ratio of stop mixing to stop mass taking its *most natural* value as defined in Eq. (8) and plotted in Fig. 1. Grey shading shows the Higgs mass uncertainty due to the top mass uncertainty. Upper panels take the top pole mass as measured by the Tevatron, ATLAS and CMS: $M_t = (173.1 \pm 0.7) \text{ GeV}$ [50]; lower panels take $M_t = (173.3 \pm 2.8)$ as extracted from the Tevatron's $\sigma(pp \rightarrow t\bar{t} + X)$ measurement [51]. The left (right) panels are for $\tan \beta = 8$ (30).

Fig. 3; the choice of error in M_t has a striking effect on the Higgs mass uncertainty.

The original motivation for this work was to see to what extent a given Higgs mass, with stop mixing set by maximal naturalness and necessarily for a given value of $\tan\beta$, could give information on the average stop mass. (A believer in $m_{\tilde{Q}_3}^2 = m_{\tilde{u}_3}^2$ would furthermore have the associated stop mass eigenvalues.) Considering only experimental uncertainty – that in M_t and m_h – we can see what range of stop masses are consistent with both ΔM_t and Δm_h from the overlap of dashed lines in Fig. 3. Taking the optimistic ΔM_t (upper panels), we see an unhelpfully large range of possible stop masses at $\tan\beta = 30$ and no possible stop masses at $\tan\beta = 8$, though the latter results simply from missing higher order terms: $M_S \gtrsim 1\text{TeV}$ should do the trick. Taking the conservative ΔM_t (lower panels), we see that essentially anything is possible. In addition to the experimental uncertainty one should consider the $\sim 2-3\text{ GeV}$ theoretical uncertainty in the Higgs mass calculation (even for more accurate expressions than (7) used here), the smaller contribution of other MSSM sparticles to the Higgs mass, and (if one likes) the potentially large contribution of beyond-the-MSSM particles to the Higgs mass. We see that the stop masses connected with a given Higgs mass are doomed to float through a surface of possible values in a multidimensional parameter space, rather than being nailed to anything concrete.

We consider going beyond a Leading-Log expression for $\delta m_{H_u}^2$ but relegate this discussion to the Appendix, as it is more involved though ultimately gives the same x_{natural} .

4 Discussion and Conclusion

We have argued that the usual approach of varying the average stop mass M_S inside one's own naturalness comfort zone, followed by freely choosing the stop mixing parameter $x \equiv (A_t^2/M_S^2)$ to be maximal for the Higgs mass m_h (i.e. $x = 6$), does not correspond *a priori* to finding the most natural arrangements. In the end however, it does: Lagrange constrained maximisation of m_h for a constant $\delta m_{H_u}^2$ gives $x_{\text{natural}} = 5-6$.

However, it would be incorrect to say that maximal-mixing scenarios (MMSs) are more natural than no-mixing scenarios (NMSs) because of the lower stop masses that are possible for the same m_h . For a given stop mass, MMSs have $\delta m_{H_u}^2$ four times larger than NMSs; the latter ought to be allowed to take their stops twice as heavy to have a fair chance at reaching the same m_h . However the fact that x_{natural} is so much closer to 6 than 0 means they will not succeed, and *this* is the reason NMSs such as low-scale gauge⁷- and anomaly-mediated SUSY breaking are disfavoured. Exclusion is certainly too strong a description of this situation, as such models may be pushed continuously into the split or high-scale SUSY regimes, with a Higgs mass that is acceptable or even too high. They merely incur a higher fine-tuning penalty, and each individual SUSY fan must to choose their own fine-tuning value (and method of quantifying it) at which to jump ship. On the other hand, the (non-)discoverability of stops at the LHC could be considered an alternative criterion for the relative merit in studying different models, which certainly favours MMSs whatever their level of A_t -induced unnaturalness.

While the average stop mass may be incomplete as a yard stick for naturalness, low $m_{\tilde{t}_1}$ gives no clue at all. Two models may have the same A_t and $m_{\tilde{Q}_3}^2 + m_{\tilde{u}_3}^2$ – and thus be equally natural – while having very different $|m_{\tilde{Q}_3}^2 - m_{\tilde{u}_3}^2|$.

For completeness we point out that in passing beyond the MSSM it cannot be taken for granted that the average stop mass gives some indication of naturalness in the same way, or at all. The trigger of electroweak symmetry breaking may no longer be the stop-induced corrections to $\delta m_{H_u}^2$ pushing $m_{H_u}^2$ sufficiently negative to outweigh the (roughly scale-independent) $|\mu|^2$ term. The classic example is the \mathbb{Z}_3 -symmetric NMSSM, where an effective μ term is itself

⁷ The more complex, $A_t(\Lambda) \neq 0$ model-building of [53] and [54] aside, that is.

induced by radiative corrections (see [55] for a review and original references). Indeed in [54] the NMSSM couplings were found to be more important for fine-tuning than the stop masses.

We have commented on the difficulty in tying down stop masses to a given Higgs mass, even invoking naturalness as a further constraining criterion. The best chance then lies with specially designed searches for stops confirming their presence or absence at the weak scale. We hope (against hope?) for the former.

Acknowledgements

Thanks to the other participants of the “Implications of a 125 GeV Higgs boson” workshop held at LPSC Grenoble, where this idea was born, and to the Centre de Physique Théorique de Grenoble for hospitality. Thanks to Carlos Wagner and Thomas Rizzo for useful communications, and to Matt Dolan, Joerg Jaeckel, Valya Khoze and Daniel Maitre for helpful comments. This work was supported by the STFC.

Appendix Beyond Leading-Log for $\delta m_{H_u}^2$

The LL expression for $\delta m_{H_u}^2$ is obtained by ignoring the scale dependence of A_t and M_S ; to do better we can consider integrating A_t and M_S over their varying higher-scale values. First consider the running of A_t and M_S with arbitrary self- and mutual-couplings as well couplings to other particles:

$$\frac{d}{dt} \begin{pmatrix} M_S^2(t) \\ A_t^2(t) \end{pmatrix} = \begin{pmatrix} a(t) & b(t) \\ 0 & c(t) \end{pmatrix} \begin{pmatrix} M_S^2(t) \\ A_t^2(t) \end{pmatrix} + \text{other running soft-mass terms} \quad (10)$$

$$\therefore \begin{pmatrix} M_S^2(t) \\ A_t^2(t) \end{pmatrix} = \begin{pmatrix} d(t) & e(t) \\ 0 & f(t) \end{pmatrix} \begin{pmatrix} M_S^2(0) \\ A_t^2(0) \end{pmatrix} + \text{other high-scale soft-mass terms}, \quad (11)$$

where a, b, c are running couplings and d, e, f are related to the former by integration; and the lower-left entry of the matrix must vanish since A_t appears in the Lagrangian, not A_t^2 . Note that if the *other* soft-mass parameters themselves run due to A_t and M_S , this feeds back into Eq. (11) as corrections to the coefficients d, e, f at the next order in $g^2/16\pi^2$, which could have an impact but we neglect this for simplicity. Integrating the $m_{H_u}^2$ beta function (2), keeping just the stop-sector terms as before but now including their scale-dependence (and that of the top Yukawa) as in (11), we have

$$\delta m_{H_u}^2(t) = \frac{3}{8\pi^2} \left(2M_S^2(0) \int_{t'=0}^{t'=t} y_t^2(t') d(t') dt' + A_t^2(0) \int_{t'=0}^{t'=t} y_t^2(t') (2e(t') + f(t')) dt' \right) + \text{other high-scale soft-mass terms} \quad (12)$$

For Lagrange constrained maximisation, Eq. (5), we must differentiate $\delta m_{H_u}^2$ and δm_h^2 with respect to A_t^2 and M_S^2 . One can either invert Eq. (11) and substitute into Eq. (12) to obtain $\delta m_{H_u}^2$ instead as a function of the *low*-scale stop parameters, or all derivatives can be taken with respect to the *high*-scale stop parameters (using the chain rule for δm_h^2 , whose arguments should be evaluated at the low scale). Both approaches give the same result, as they must:

$$f(t) \left(2 \int_{t'=0}^{t'=t} y_t^2(t') d(t') dt' + d(t) y_t^2(t) \left(1 + \frac{A_t^2(t)}{2M_S^2(t)} \right) \right) \frac{\partial(\delta m_h^2)}{\partial A_t^2} = \left(d(t) \int_{t'=0}^{t'=t} y_t^2(t') (2e(t') + f(t')) dt' - 2e(t) \int_{t'=0}^{t'=t} y_t^2(t') d(t') dt' \right) \frac{\partial(\delta m_h^2)}{\partial M_S^2} \quad (13)$$

The Leading-Log relation is recovered for $(d(t), e(t), f(t)) = (1, 0, 1)$, $y_t(t) = y_t$. So what are these functions $d(t), e(t), f(t)$ in the MSSM? Expressions for the one-loop running parameters can be written down when all Yukawa couplings except that of the top are set to zero [56–58]. y_t and $A_t(t)$ do not require numerical integration if one also sets the $U(1)$ and $SU(2)$ gauge couplings to zero: one finds

$$y_t^2(t) = y_t^2(0) \xi^{-16/9}(t) G^{-1}(t; \frac{-16}{9}) \quad (14)$$

$$A_t(t) = G^{-1}(t; \frac{-16}{9}) \left[A_t(0) + \frac{16}{9} M_3(0) \left(G(t; \frac{-16}{9}) \xi^{-1}(t) - G(t; \frac{-25}{9}) \right) \right] \quad (15)$$

$$\text{where } \xi(t) = 1 + \frac{3}{2\pi} \alpha_3(0)t$$

$$G(t; n) = 1 - \frac{3}{4\pi^2} y_t^2(0) \int_0^t dt' \xi^n(t')$$

From (15) we read off that $f(t) = G^{-1}(t; \frac{-16}{9})$. In this same scheme for extracting running parameters, the stop mass necessarily involves numerical integration. However RG-induced splitting of the stop from the lighter generation up-type quarks is typically small (and if not the model is generically unnatural, as mentioned in the introduction), so that the running stop is well-approximated by its high-scale value plus the gluino-induced term, the latter easily obtained by integrating the one-loop running gluino mass:

$$M_S^2(t) = M_S^2(0) + \frac{8}{9} M_3^2(0) \left(\frac{\alpha_3^2(t)}{\alpha_3^2(0)} - 1 \right) \quad (16)$$

This gives $d(t) = 1$, $e(t) = 0$. Eq. (13) is then

$$\begin{aligned} 2G^{-1}(t; \frac{-16}{9}) \left(\int_{t'=0}^{t'=t} \left(\xi^{-16/9}(t') G^{-1}(t'; \frac{-16}{9}) \right) dt' \right. \\ \left. + \xi^{-16/9}(t) G^{-1}(t; \frac{-16}{9}) \left(1 + \frac{A_t^2(t)}{2M_S^2(t)} \right) \right) \frac{\partial(\delta m_h^2)}{\partial A_t^2} = \\ \int_{t'=0}^{t'=t} \left(\xi^{-16/9}(t') G^{-2}(t'; \frac{-16}{9}) \right) dt' \frac{\partial(\delta m_h^2)}{\partial M_S^2} \quad (17) \end{aligned}$$

The integrals can be done analytically and the resulting root, x_{natural} , found; we do not plot it as it is essentially indistinguishable from the one shown in Fig. 3, even for very high mediation scales $\Lambda \sim 10^{16}$ GeV. Thus our attempt at an approximate RG improvement of $\delta m_{H_u}^2$ (resumming all the logs that come with appreciable coupling constant factors) makes no difference to the result obtained from the leading log expression.

An alternative approach to this approximate RG improvement would be to work consistently at Next-to-Leading-Log NLL order for $\delta m_{H_u}^2$. Barbieri-Giudice fine-tuning measures are given at NLL in [38], from which one can extract the dependence of $\delta m_{H_u}^2$ on any trilinear mixing term or sfermion mass-squared via

$$\delta m_{H_u}^2(A_i) = \int \left(\frac{M_Z^2}{2A_i} Z_{A_i} \Big|_{\tan \beta \rightarrow \infty} \right) dA_i \quad (18)$$

$$\delta m_{H_u}^2(m_{\tilde{f}}^2) = \frac{M_Z^2}{4} Z_{m_{\tilde{f}}} \Big|_{\tan \beta \rightarrow \infty} \quad (19)$$

$$\text{where } Z_{p_i} \equiv \frac{\partial(\log M_Z^2)}{\partial(\log p_i)}$$

Note that $\delta m_{H_u}^2$ and $m_{\tilde{f}}^2$ both having mass dimension 2 results in $Z_{m_{\tilde{f}}} \propto m_{\tilde{f}}$, giving the simpler expression (19). Z_{A_i} however can contain further mass-scales beyond A_i ; indeed for the stop, Z_{A_t} contains terms with M_1 , M_3 and A_b . In other words, at NLL $\delta m_{H_u}^2$ depends on A_t not only through an A_t^2 term but also through terms $A_t M_1$, $A_t M_3$ and $A_t A_b$. In the spirit of connecting $\delta m_{H_u}^2$ and the Higgs mass to the stop sector in isolation, we will not explore this effect here.

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